# Comparative Analysis of Phase Retrieval and Shack-Hartmann Wavefront Sensing Space Based Segmented Optical Control System

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#### <u>Abstract</u>

We present an ongoing study to compare various methods of **phase retrieval**, **phase diversity**, and **Shack-Hartmann** sensing for *fine figure control* of deployable segmented aperture space optical systems. We develop comphrehensive models of an optical system, active optical bench, focal plane, phase retrieval algorithms, control algorithms and compare the various methods in a Monte-Carlo type fashion. The results will eventually be compared with actual results from a NASA testbed and flight missions.



CESDIS

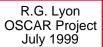
Viewgraphs will be available at http://jansky.gsfc.nasa.gov/OSCAR



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- Summary and Conclusions







# Study Goals and Methodology

#### • GOALs:

- "BEST" WFS / Optical Control or Segm't Aper Space Telescopes.
- Develop / Verify hi-fidelity optical model of entire control process.
- model useful for many NASA projects.

#### "best" refers to:

accuracy/precision, spatial frequency image quality, dynamic range robustness, computational complexity

#### Methodology

- develop high fidelity computer model of entire control loop
   OTA, active bench, focal plane, WFS, actuators, control alg.
- include effects of:
  - readnoise, darkcurrent noise, photon noise, sampling polychromatism, jitter, sampling, pixelization, detector MTF mid- and hi-spatial freq, micro-roughness, actuator influence
- cycle model in Monte-Carlo fashion, tabulate and compare results
- verify as "real" data becomes available
- update model





# **Optical Modelling**

- Initial Study "tuned" to Developmental Comparative Active Telescope Testbed (DCATT)
- includes:
  - Full-Aperture Zernikes /Sub-Aperture Zernikes, {0 2.5λ rms}.
  - 90.3 cm aperture, 21% obscured, 7 hex segments.
  - Residual Polish Marks / Random power law surfs.
  - Polychromatic PSFs,  $\{\lambda=0.6328 \text{ um}, \Delta\lambda=10\text{nm}\}.$
  - System Jitter, {0 1.5 pixels rms}.
  - Pixelization {9 um pixels}.
  - read noise {13 electrons rms}.
  - photon noise.
  - full-well {80,000 e}.
  - quantization {12 bit = 4096}.
  - Actuator Influence functions (Xinetics 349 DM)
- Software developed on MasPar MP2 (16,384 procs / 6 GFLOPs)
- NASA

Currently being ported to C/MPI



#### Example of LEO System Control File

#### **Keywords for:**

- Pupil diameters, wavelengths, bandpass,
- pixelization, obscuration ratio,
- full aperture Zernikes, spiders, pads,
- aper rotation with respect to CCD line/scan
- jitter in x and y directions,
- PM and SM secondary mirror mid-spatial
- random high spatial frequencies
- detector dynamic range, shot noise,
- readnoise, quantization, finite pixel size.
- actuator input files, influence functions

= aper_22.in	# Aperture control file (set to "none" if not using). R.G. Lyo
= 0.903	# Exit pupil Diameter (meters) (system is F/15) OSCAR Pr
= 0.6328	# Wavelength (microns) July 199
= 0.1330311	# Output Sample spacing (arseconds) (pixels are 9 um)
= Y	# Generate PRF (Y or N)
= 0.1330311	# Detector element size for PRF (arseconds)
Npsf= 7	# Number of PSF's across Passband.
fwhm= 0.0100	# FWHM of Filter (microns) (if Npsf's > 1 )
= 0.21384	# aperture obscuration ratio (0.21384)
Ztype= 1	# Zernike type (0 = Annular, 1 = Code 5)
Z01= -0.051605	# Piston (microns WFE )
Z02= 0.014292	# X-tilt (microns WFE )
Z03= 0.121884	# Y-tilt (microns WFE )
Z04= 0.012207	# X-Y astigmatism (microns WFE )
Z05= 0.121851	# Focus (microns WFE )
Z06= 0.164621	# 45-degree (microns WFE )
Z07= 0.187863	# Trefoil (microns WFE )
Z08= 0.012174	# X-coma (microns WFE )
Z09= -0.110907	# Y-coma(microns WFE )
Z10= -0.090205	# Trefoil (microns WFE )
Z11= -0.126233	# (microns WFE )
Z12= 0.227831	# (microns WFE )
Z13= 0.078349 Z14= -0.084945	# Fourth-order spherical (microns WFE )
214= -0.084945	# (microns WFE )
•	
Z32= -0.144799	# (microns WFE )
= N	# Are OTA SMA spiders present? (Y or N)
= N	# Are OTA PMA pads present? (Y or N)
= 0.000000	# OTA aperture rotation angle (degrees)
= 0.000000	# WF/PC aperture rotation angle (degrees)
= 1	# Computed PSF resampling factor (integer > 0)
= 0.000000	# WF/PC aperture X-axis offset (pixels)
= 0.000000	# WF/PC aperture Y-axis offset (pixels)
= N	# Use PM surface map? (Y or N)
= N	# Use SM surface map? (Y or N)
= 0.0	# x - Jitter (milli-arcseconds)
= 0.0	# y - Jitter (milli-arcseconds)
= Y	# Create phase map file ?(Y or N)
= N	# Use Recovered Surface Map (Y/N/P), if Y or P then PM=SM=N.
= N	# Apodized the Pupil function ? (Y or N)
= N	# Add in random Gaussian surface (Y or N)
= 0.01	# S. Dev. of random Gaussian surface (microns).
= 993	# integer seed value for random surface generator.
= act0.in	# Actuator file ("none" if not using)(units are um WFE).
Nact= 349	# Total number actuators in pupil.

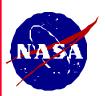
# R(r) = exp(-act\_cof\*r)\*sin(act\_cof\*r+PI/4)

# number of quantize levels (4096 levels, 0=>not quantize).

# Detector fullwell in electrons.

# Detector readnoise in electrons.

# Add in shot noise (Y/N).



8/11/99

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act cof= 39.8175

fullwell = 80000.0

readnoise = 13.0

quant = 4096.0

shot = Y



#### R.G. Lyon OSCAR Project July 1999

#### **Keywords for:**

•POLY => regular polygon apertures

•TRIA => triangular apertures

•RECT => rectangular apertures

• CIRC => circular apertures

- each aperture can have it;s own Zernikes, or can AND'ed with other sub-aps to form a segment.
- Each set of Zernikes can have it's own center and normalization radius.

```
POLY = 1 {
        Nsides = 6
                                # Number of sides of polygon
        radius = 0.173205
                                # radius of inscribed polygon (meters)
        xcent = 0.000000
                                # X-center of polygon (meters)
        ycent = 0.000000
                                # Y-center of polygon (meters)
        theta
              = 0.000000
                                # rotation angle of polygon (degrees))
        piston = 0.000000
                                # piston error (microns-surface error)
               = 0.000000
                                # tilt of mirror in x-direction (arcsec)
               = 0.000000
                                # tilt of mirror in y-direction (arcsec)
               = -0.041284
                                # Piston (microns WFE)
               = 0.011433
                               # X-tilt (microns WFE)
               = 0.097507
                                # Y-tilt (microns WFE)
               = 0.009765
                                # X-Y astigmatism (microns WFE)
               = 0.097481
                                # Focus (microns WFE)
               = 0.131696
                                # 45-degree astigmatism (microns WFE)
                                # Trefoil (microns WFE)
               = 0.150290
               = 0.009740
                                # X-coma (microns WFE)
               = -0.088725
                                # Y-coma (microns WFE)
               = -0.072165
                                # Trefoil (microns WFE)
        Z11
               = -0.100985
                                # (microns WFE)
        Z12
               = 0.182264
                                # (microns WFE)
               = 0.062679
                                # Fourth-order spherical (microns WFE)
        apodize = N
                                # anti-alias mask (Y/N).
POLY = 2 {
        Nsides = 6
                                # Number of sides of polygon
        radius = 0.173205
                                # radius of inscribed polygon (meters)
        xcent = 0.000000
                                # X-center of polygon (meters)
        ycent = -0.306000
                                # Y-center of polygon (meters)
        theta = 0.000000
                                # rotation angle of polygon (degrees))
        piston = 0.000000
                                # piston error (microns-surface error)
               = 0.000000
                                # tilt of mirror in x-direction (arcsec)
               = 0.000000
                                # tilt of mirror in y-direction (arcsec)
               = -0.115865
                                # Piston (microns WFE)
               = 0.044346
                                # X-tilt (microns WFE)
               = -0.110537
                                # Y-tilt (microns WFE)
               = 0.020632
                                # X-Y astigmatism (microns WFE)
               = 0.043695
                                # Focus (microns WFE)
               = 0.113846
                                # 45-degree astigmatism (microns WFE)
                               # Trefoil (microns WFE)
               = 0.157808
               = 0.087005
                                # X-coma (microns WFE)
               = 0.209875
                                # Y-coma (microns WFE)
               = -0.004463
                                # Trefoil (microns WFE)
               = -0.097702
                                # (microns WFE)
               = -0.120061
                                # (microns WFE)
               = -0.013088
                                # Fourth-order spherical (microns WFE)
                                # anti-alias mask (Y/N).
        apodize = N
```

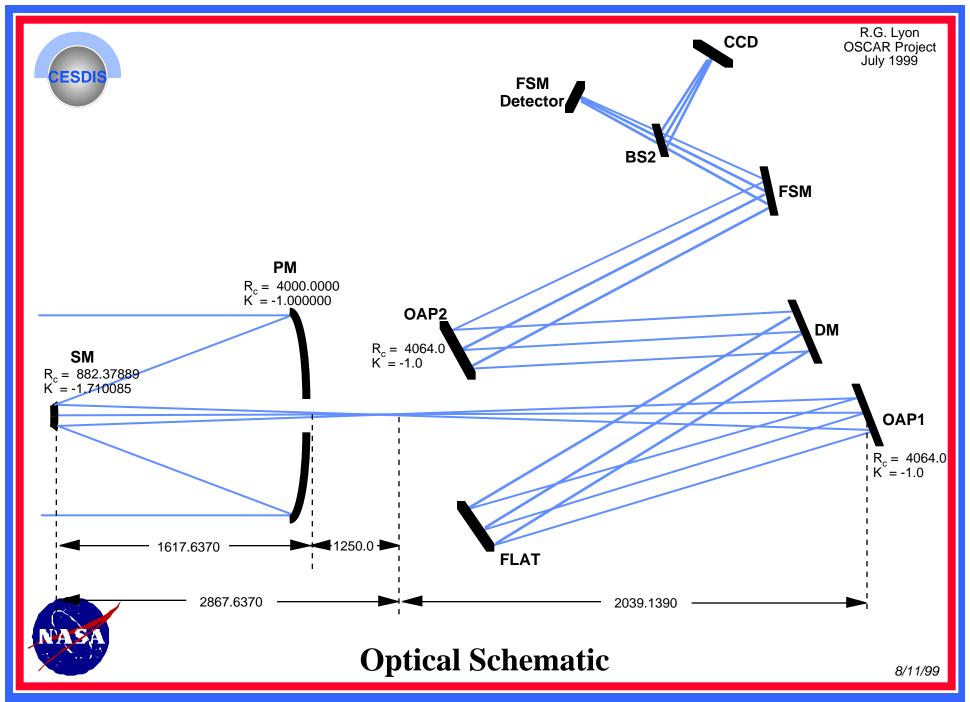




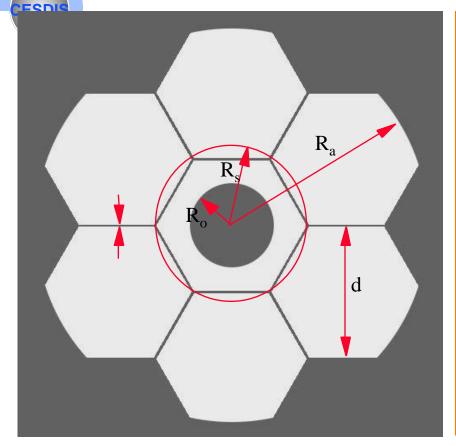
# **Optical Modeling**

- (LEO) Lyon's Electro-Optical Modeling and Analysis Software
  - Multiple plane diffraction, Fresnel, Fraunhoffer and rigorous Angular Spectrum.
  - Segmented apertures and deformable mirrors, influence functions, range limits, clamped and floating actuator models.
  - Ful- and sub-aperture Zernike polynomials.
  - power law random surfaces.
  - White noise, harmonic and low frequency jitter models.
  - Detector effects, MTF, charge transfer efficiency, pixelization effects, quantization error, dynamic range effects, quantum efficiency.
  - Gaussian and Poisson noise models.
  - System radiometry spectral filter functions, optics transmission.
  - Coronagraph capability with assortment of masks and Lyot stops.
  - Scattering, Surface Scatter, Diamond Turning, Atmosphere
  - Scene Modeling Fractal landmass, cloud and water models from LEO/ GEO and with scan mirror options.
  - Fourier Transform based Imaging Interferometer model.
  - Inhomogenous wave propagation finite element model (in progress)
  - Shack-Hartmann sensor model (in progress).

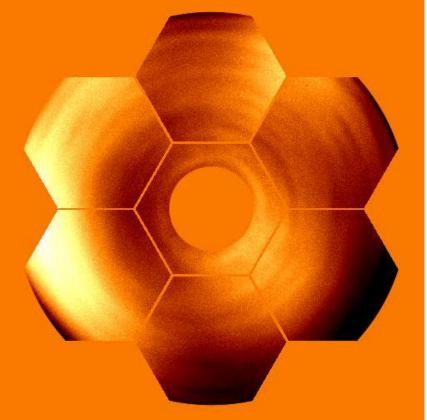




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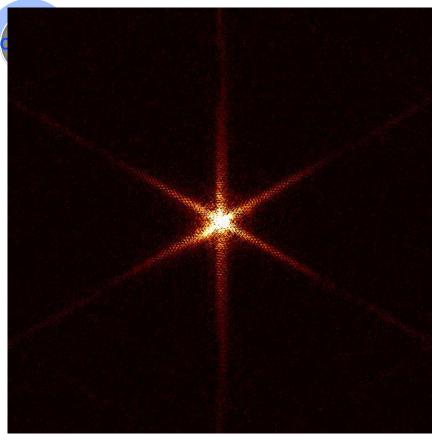


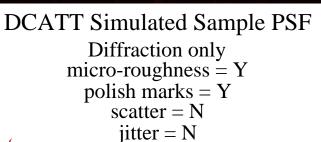
DCATT Aperture Function Geometry d = 300.000 mm., = 6.00 mm.  $R_a = 451.690 \text{ mm.}, R_s = 173.205 \text{ mm.}$  $R_o = 96.589 \text{ mm.}$ 

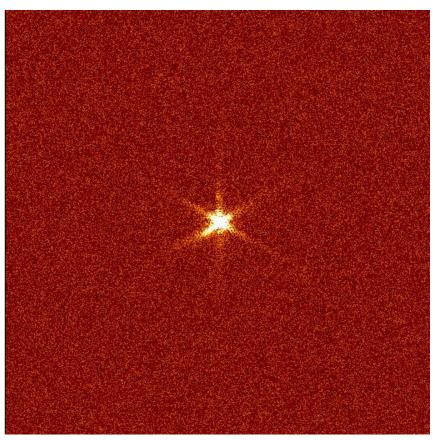


Sample DCATT Phase Function = 0.285 max = 0.847 min = -1.327

## **Simulated Pupil and Phase Function**







DCATT Simulated Sample PRF

12bit = 4096

fullwell = 80,000 e

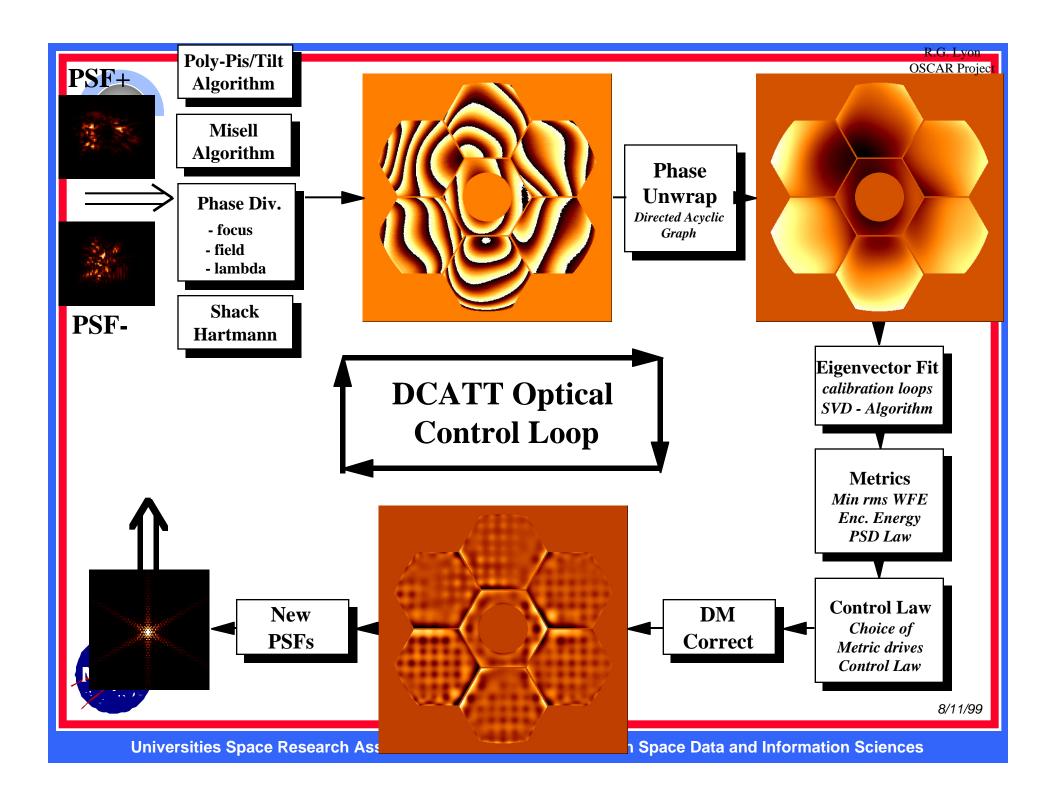
readnoise = 13 e rms

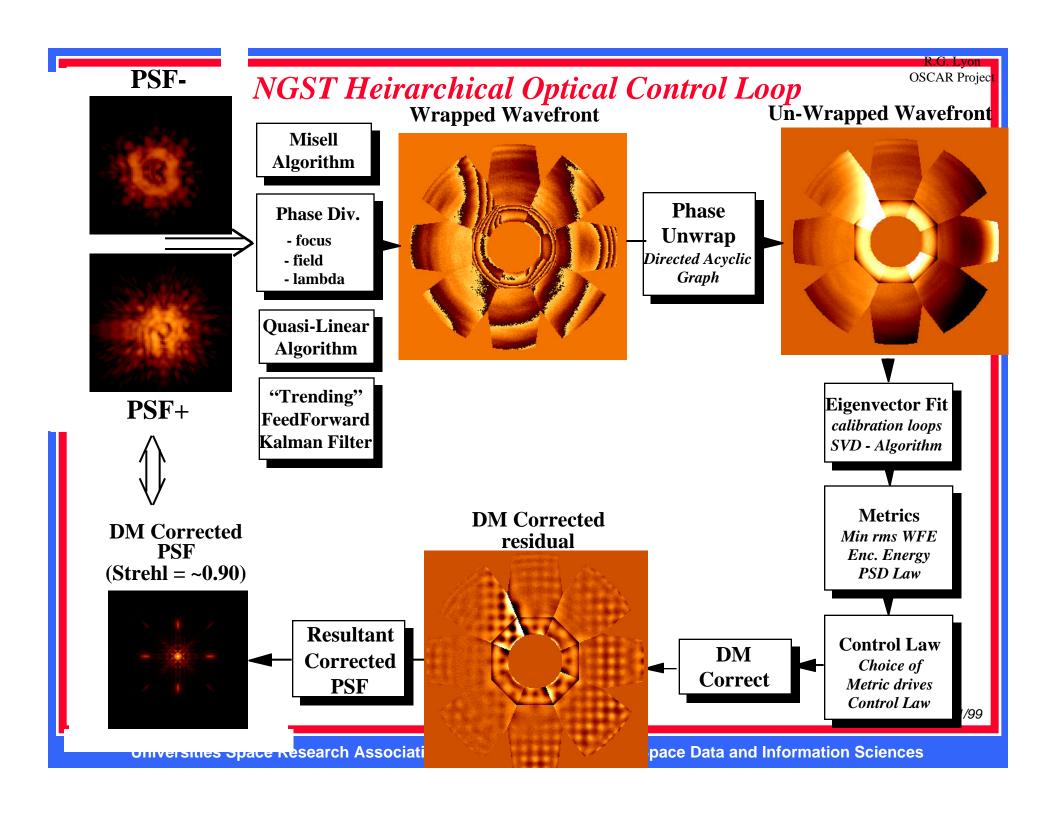
shot noise = Y

detectors = 9 µm



### **Simulated Point Spread Function and Point Response Function**







### **Phase Retrieval Problem Statement**

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- From observed *Point Response Functions* find wavefront error (WFE)
- WFE gives optical surface structure.
- Point Spread Function is modulus squared of 2D Fourier Transform of complex pupil function. Image = convolution of PSF with object.

$$PSF(x, y; \alpha, \lambda) = \begin{vmatrix} \frac{1}{\lambda F} & A(u, v) e^{i\phi(u, v; \alpha)} & e^{-i2\pi(f_x u + f_y v)} & \text{du dv} \end{vmatrix}^{2}$$

$$f_x = \frac{-x}{\lambda F} \quad \text{and} \quad f_y = \frac{-x}{\lambda F}$$

 The PRF is PSF integrated across spectral passband, convolved with detector spatial response and sampled.

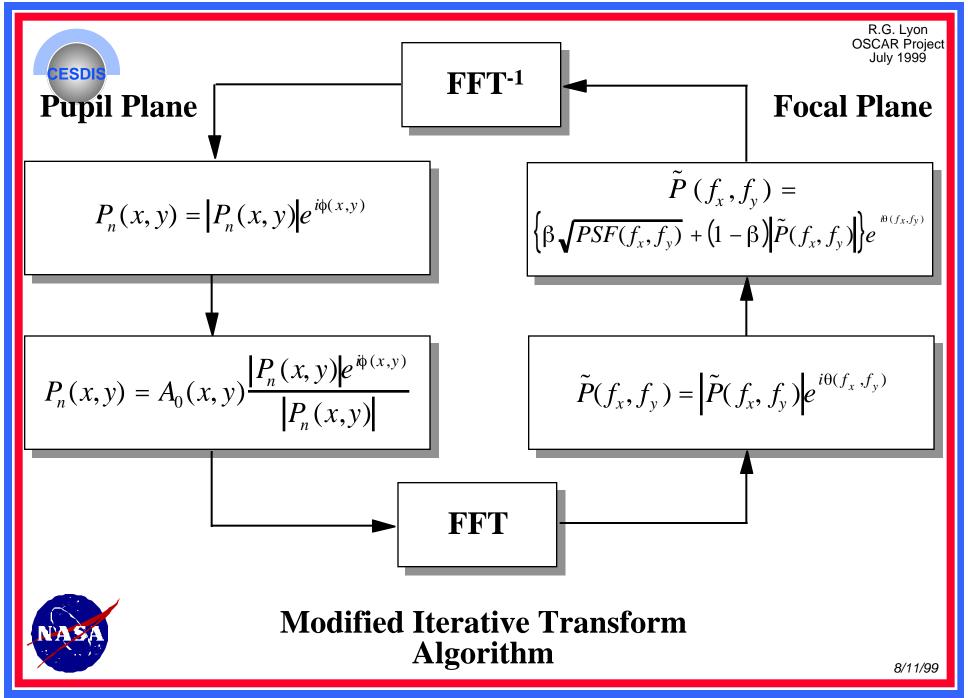
$$PRF(x, y; \alpha) = PSF(x, y; \alpha, \lambda) S(\lambda) d\lambda ** rect(\frac{x}{x}) rect(\frac{y}{y})$$

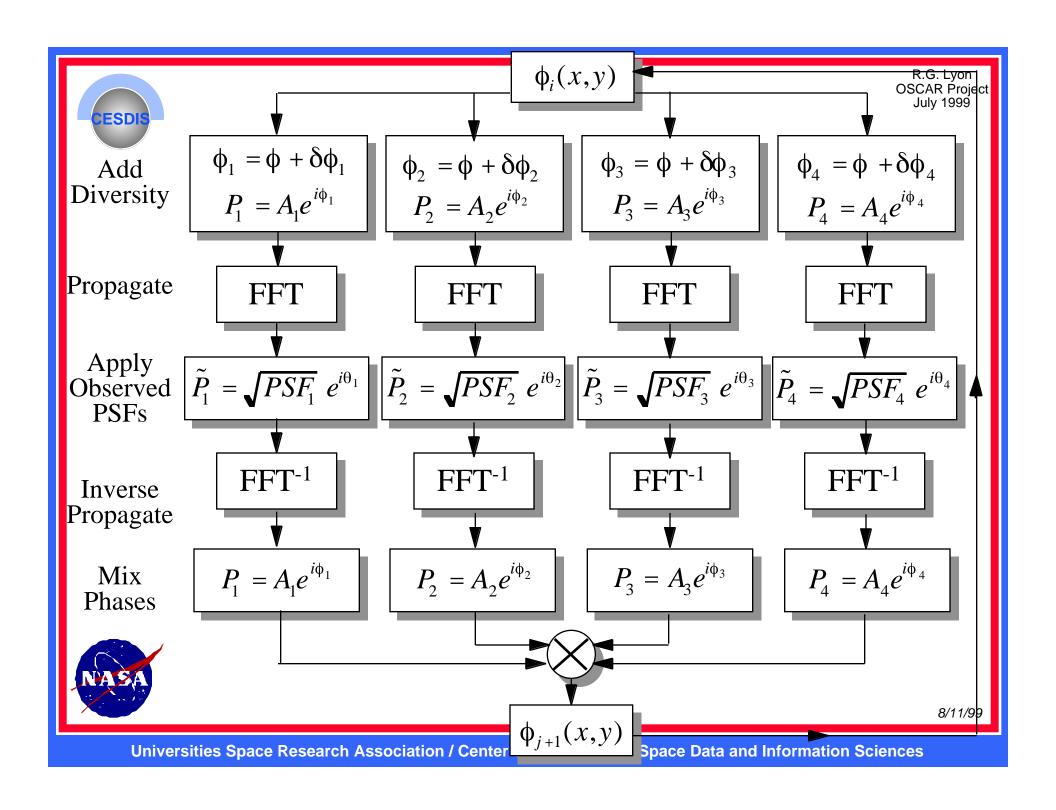
What's actually measured is given by:

$$M(x, y; \alpha, flux, A, B, C) = flux * PRF(x, y; \alpha) + A * x + B * y + C + \eta(x, y)$$



 Requires complex nonlinear iterative algorithms to find (u,v) from M(x,y, , flux,A,B,C)



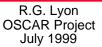




#### **Control Precision vs. rms WFE**

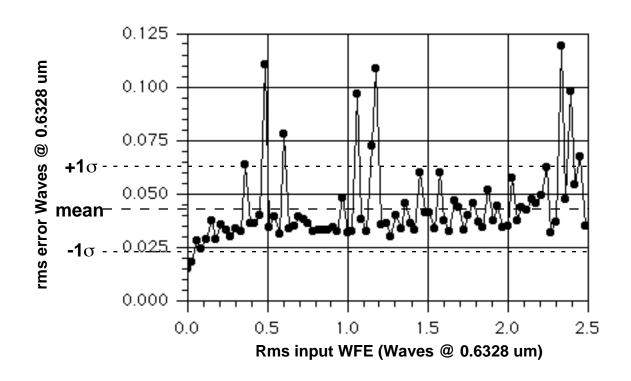
- Utilized LEO to generate 400 PSFs, 4 each at 100 realizations of WFE from 0 - 2.5λ rms.
  - Full aperture Zernikes, Sub-aperture Zernikes, segment piston/tip/tilt
  - finite spectral bandpass (=0.6328, =10mm)
  - detector and actuator effects.
- Passed each through:
  - 4 PSF Misell,
  - phase unwrapping,
  - DM control, DM control with limits,
  - DM+PM control.
- Tabulated output precisions vs. rms WFE for each case.
- WFS precision is  $\lambda/23$  +/-  $\lambda/52$  rms
- Control with range limited DM+PM ~λ/10
- RSS of Sensing and Control errors is ~λ/9







## **Wavefront Sensing Precision**



- Wavefront sensing precision is  $\lambda/23 + \lambda/52$  over range  $\{0 2.5\lambda\}$ .
- Anomolies caused by phase unwrapping problems.

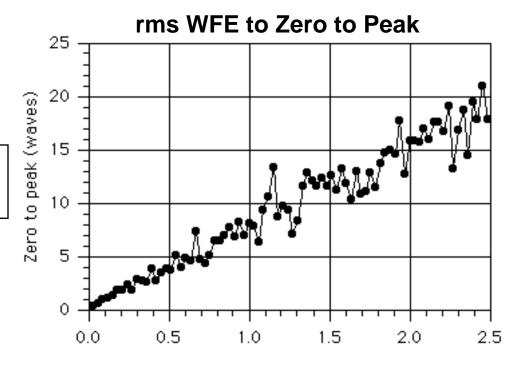




# Phase Unwrapping

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slope ~7.5 waves/rms wave phase unwrap for > 0.15 waves rms



rms WFE (waves @lambda = 0.6328 um)

- Number of Phase Unwrapping Methods Explored
- **Problems occur for:** 

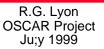
  - Large slopes => unresolved edges
  - Jitter

=> "Branch Points"

Low SNR

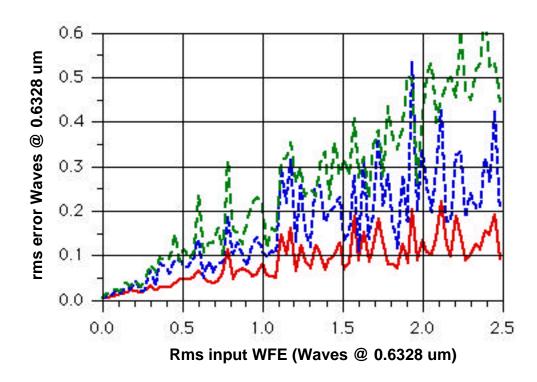
- => poor "fringe edge"
- Need diagnostic to tell if needed and if failed







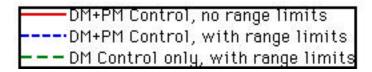
#### DM and PM Control vs rms WFE



 Over error budgeted range {0 - 1λ rms}

**DM+PM controls <**  $\lambda/10$ 

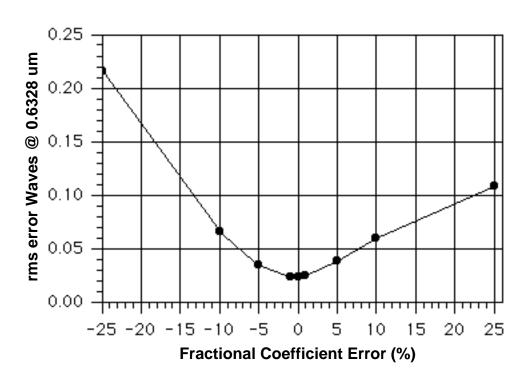
• Error grows linearly with rms WFE







### Influence Function Sensitivity



•Influence function model for DM from Dave Redding:

$$R(r) = e^{-Ar}\cos(Ar)$$



- "A" varied over +/- 25% from nominal
- Input WFE = 0.25 waves rms (lambda = 0.6328 um)



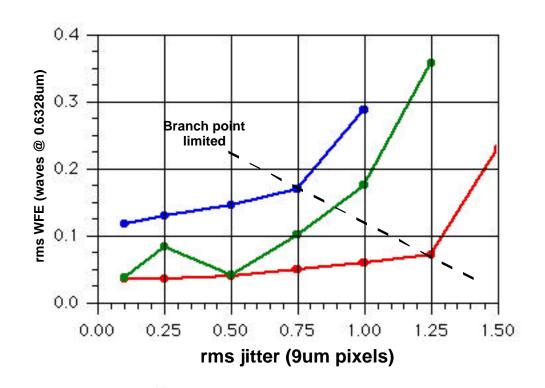
### **WFS Precision due to Jitter**

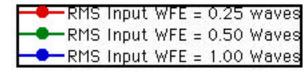
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- Jitter model is non-deterministic.
- Low-pass filter on OTF:

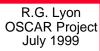
$$\langle H(f_x, f_y) \rangle = e^{-\left(\sigma_x^2 f_x^2 + \sigma_y^2 f_y^2\right)}$$

- x-rms <=> y-rms
- Hi-FI models possible.
- effect is generation of "unphysical" branch points.
- Branch points cause errors in phase unwrapping.
- Useful as diagnostic ?
- Number of branch points imply severity of jitter.
- Jitter deconvolution possible?
- Metric would be residual branch points.



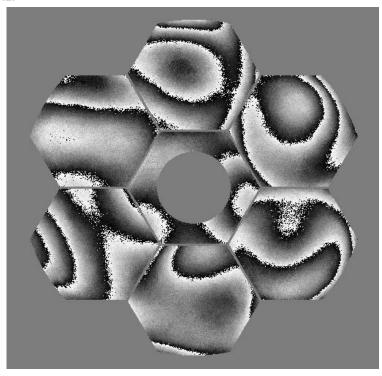




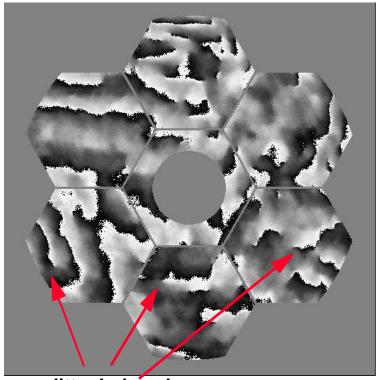


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#### **Jitter Induced Branch Points**



WFE = 1.00 waves rms Jitter = .1 pixel rms (9 um)

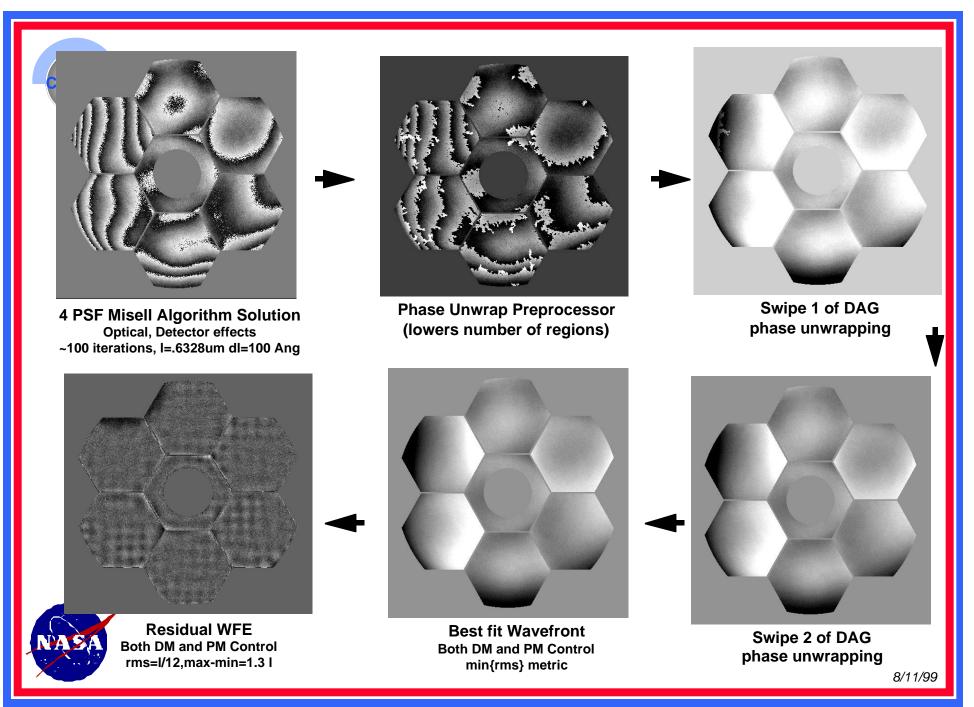


Jitter Induced Branch Points

WFE = 1.00 waves rms Jitter = 1 pixel rms (9 um)



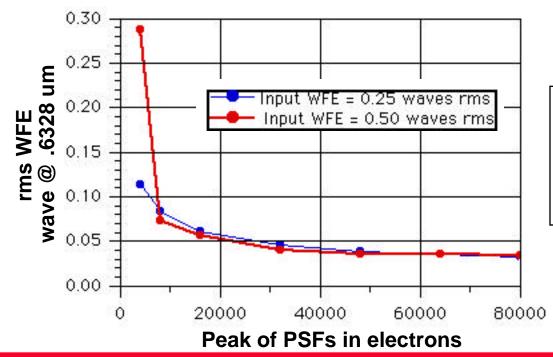
Allowable Jitter strongly correlated with WFE. Jitter makes phase unwrapping tough.





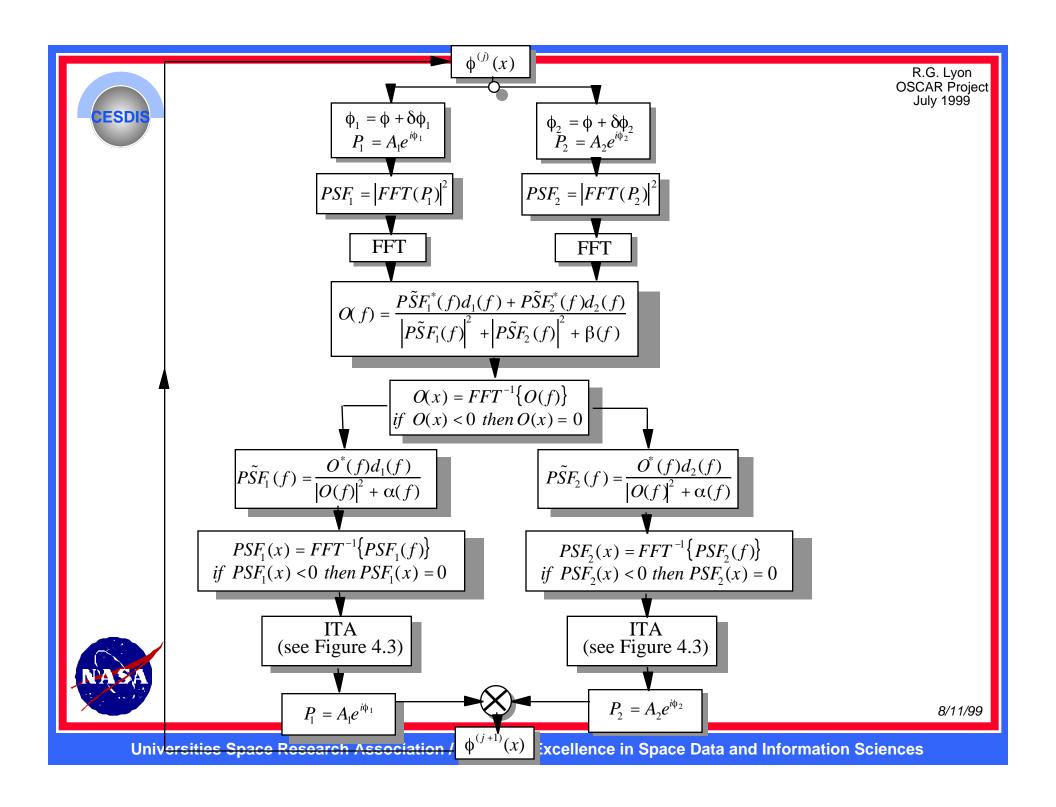
### WFS Precision vs. SNR

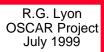
- Utilized LEO to generate sequence of different SNR PSFs,
- 7 sets at  $\sigma$ =0.25 $\lambda$  rms, and 7 sets at  $\sigma$ =0.25 $\lambda$  rms, 4 PSFs/realization.
- Passed each through 4 PSF Misell with phase unwrapping.
- Tabulated output precisions vs. SNR
- RMS output error was  $< \lambda/20 > 20,000$  electrons.
- Errors due to: phase edge loss and increased phase noise



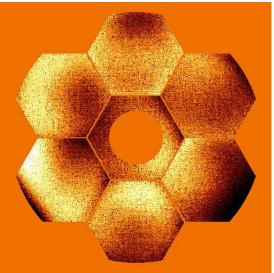
KAF1600 chip readnoise 13e rms. 80,000e full well. 12 bit quantization. 9 micron pixels



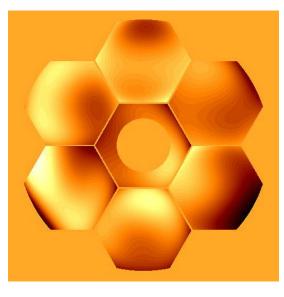




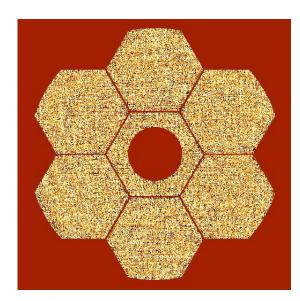




Phase Diverse Recovered Phase 0.320 λ rms

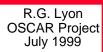


True Phase  $0.273 \lambda \text{ rms}$ 

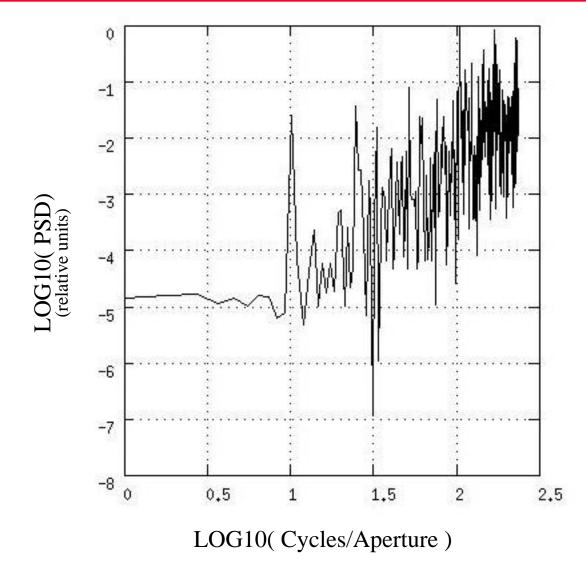


Difference Phase  $0.167 \lambda \text{ rms}$ 

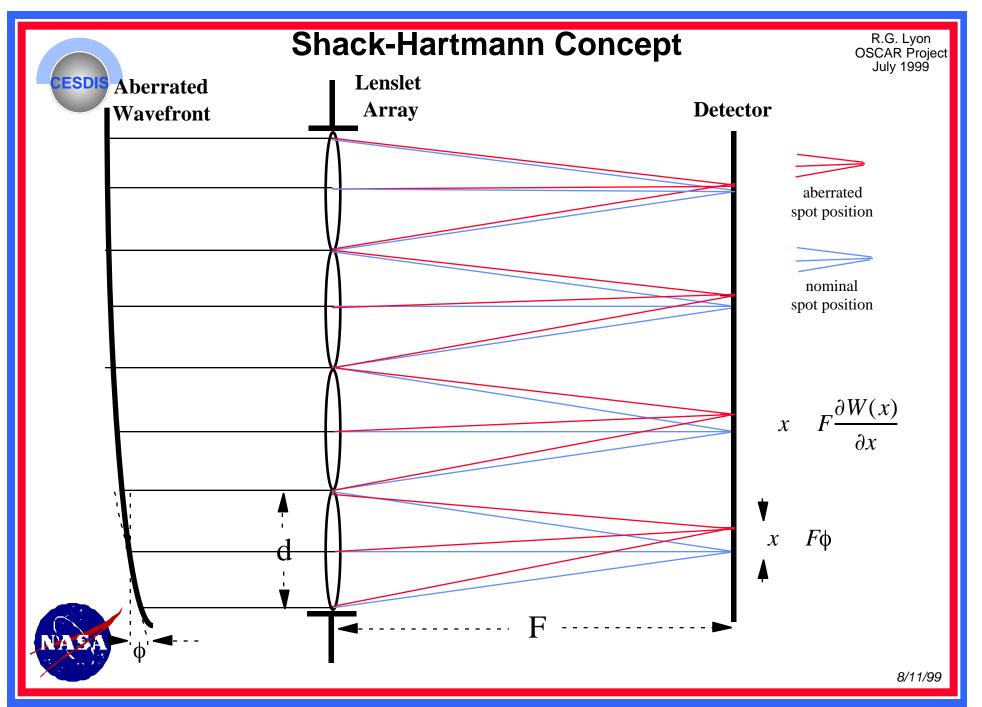


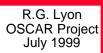




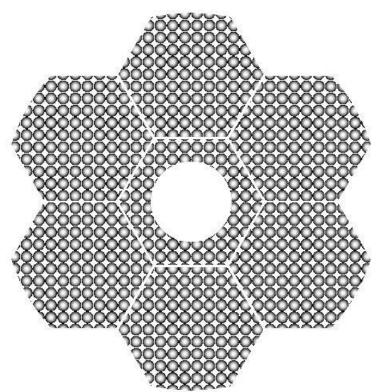


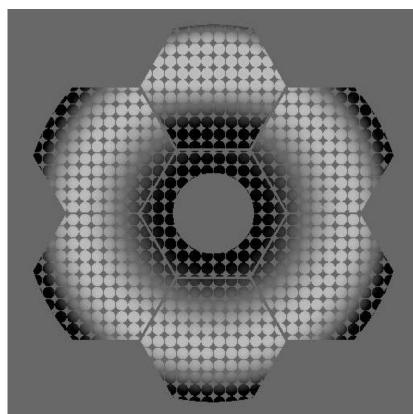




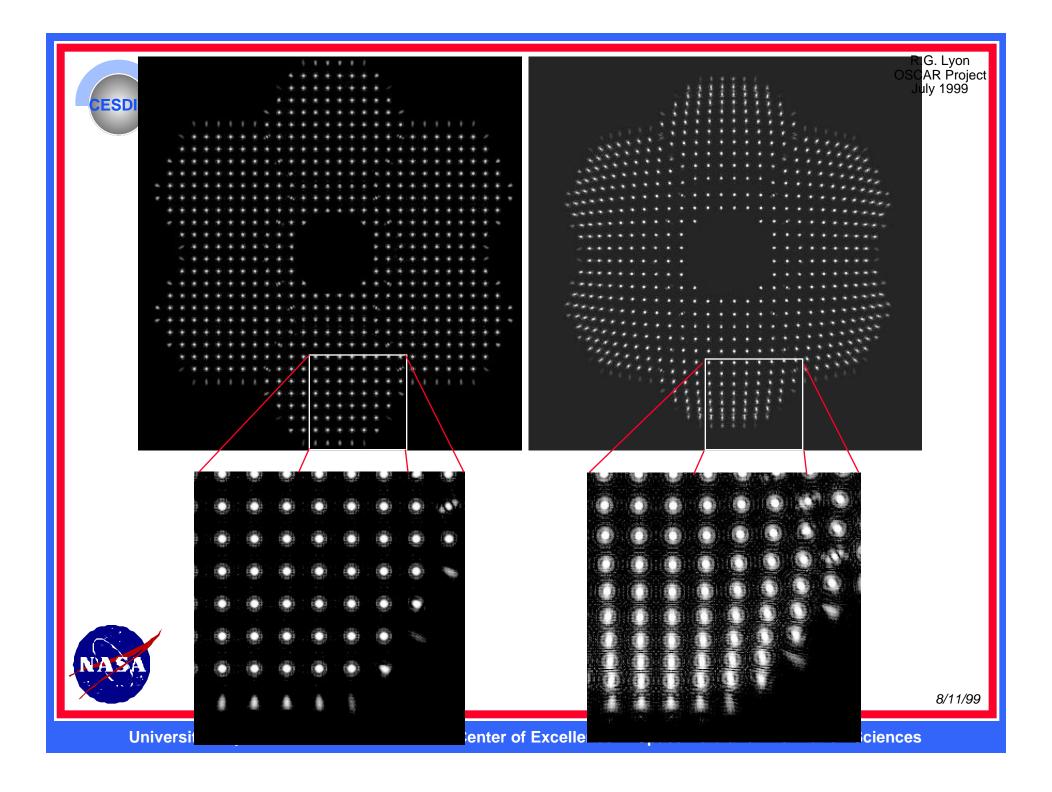


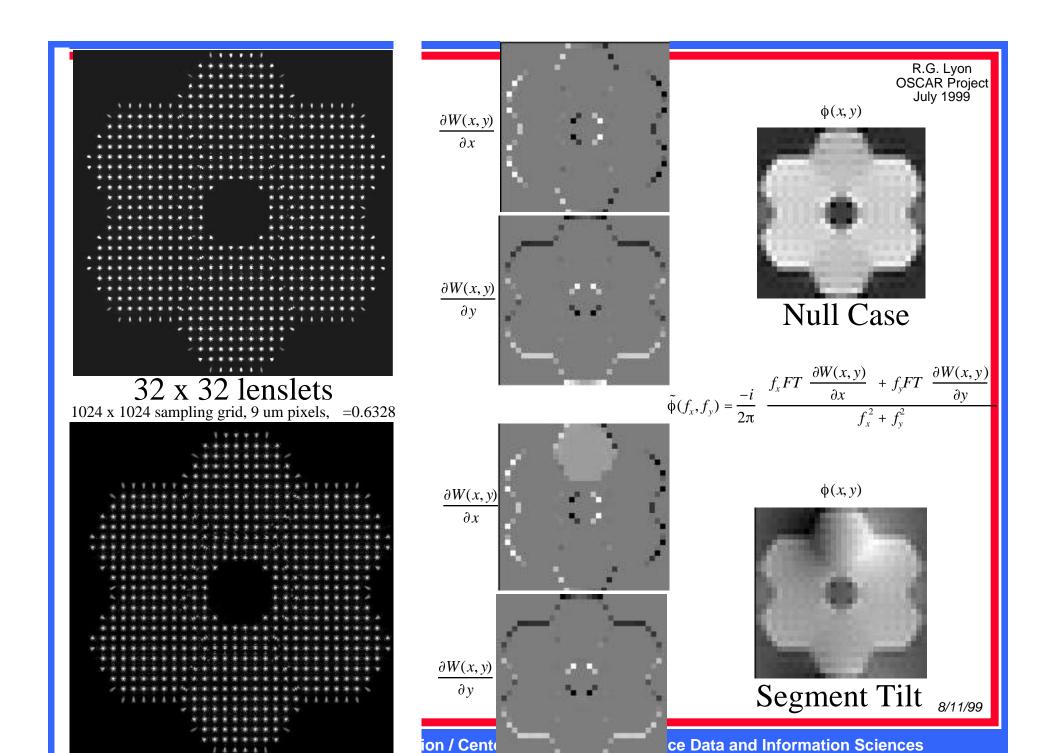


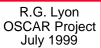














# Major Results of models for DCATT to Date

Simulations show error budget can be met for DCATT Fine Figure Control.

Over budgeted range 0 - 1  $\lambda$  rms input WFE for DCATT

WFS DM+PM Control Jitter SNR	$\sim \lambda/23$ < $\lambda/10$ < $\lambda/8$ (0.25 rms jitter) $\sim \lambda/10$ (Fullwell > 20000, SNR > 62
σ (RSS)	~λ/5.2
σ (error budget)	λ/4.43

- Doesn't include stray light, diamond turning or turbulence.
- WFS Precision strongly correlated with Jitter.
- Phase unwrapping needs to be addressed in more detail.
- SNR doesn't appear to be a problem.
- Phase Retrieval can be used over much large input WFE range.





# **Summary**

- Developed Hi-Fi model of segmented optical system.
- Developed model and simulated active optical control loop.
- Developed model and simulated Phase Retrieval.
- Developed model and simulated Shack-Hartmann.
- Studied effects of {jitter, SNR, Phase Retrieval, control...}
  - developed on MasPar MP2, in a superset of "C".
  - currently being converted to "C" with MPI.
- Still must perform studies with Shack-Hartmann
- Compare Phase Retr. vs Shack-Hartmann, other methods?
- 1st Report available on http://jansky.gsfc.nasa.gov/OSCAR

#### **Future Work**

- "Quasi-Linear" WFS, In-situ science obs WFS.
- In-situ system calibration (aka system identification).
- Hi-density actuators.
- Verify models and algorithms with "real" data.

